(6 Pages) **Reg. No. : ....**

Code No. : 6831 **Sub. Code : PMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

First Semester

Mathematics — Core

ALGEBRA — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. In a quotient group $\frac{G}{N}$, N is

   (a) any proper subgroup of G

   (b) a cyclic subgroup of G

   (c) a normal subgroup of G

   (d) a proper abelian subgroup of G

2. If $\phi : G \to \overline{G}$ is a homomorphism, then $\phi(abc) =$ ____________

(a) $\phi(a).\phi(b).\phi(c)$ (b) $\phi\left(\frac{a}{b}\right).\phi(bc)$

(c) $\phi(a) + \phi(b)\phi(c)$ (d) none of these

3. Every group is isomorphic to a subgroup of the group of automorphisms $A(S)$ for some set S is due to

(a) Legrange (b) Cayley

(c) Cayley Hamilton (d) Sylow

4. If G is a group having 36 elements and H is a subgroup with 9 elements, then $i(H)$ =

(a) 4! (b) 4

(c) 3 (d) 9!

5. Product of seven even and four odd permutations is an ____________ permutation.

(a) odd

(b) even

(c) either odd or even

(d) none

6. The group S_n has ―――――――― elements.

(a) n (b) $n!$

(c) $\frac{n!}{2}$ (d) nc_2

7. The number of 2-Sylow subgroups of order 2 in S_3 is

(a) 1 (b) 4

(c) 2 (d) 3

8. Any group of order 72 must have a normal subgroup and hence

(a) is simple (b) not simple

(c) neither (a) nor (b) (d) none of the above

9. The number of non-isomorphic abelian groups of order 3^4 is

(a) 4 (b) 3

(c) 5 (d) 6

10. Number of 3-sylow subgroup in a group of order 15 is

(a) 1 (b) 2

(c) 3 (d) 5

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Define cosets. Prove that the subgroup N of G is normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

Or

(b) Define the terms homomorphism, $Ker\phi$ and normal subgroup. Prove that if $\phi : G \to \overline{G}$ is a homomorphism, then $Ker\phi$ is a normal subgroup of G.

12. (a) Define inner automorphism. Prove that $I(G) \cong \frac{G}{Z}$, where $I(G)$ is the group of inner automorphisms of G and Z is the centre of G.

Or

(b) What are the elements in the group of automorphisms of an infinite cyclic group?

13. (a) Define conjugacy and prove that conjugacy is an equivalence relation on G.

Or

(b) Define normalizer of an element in a group. Prove that it is a subgroup of G.

14. (a) If A and B are finite subgroup of G then prove that $o(A \times B) = \dfrac{o(A)o(B)}{o(A \cap \times Bx^{-1})}$.

Or

(b) Prove that any group of order 72 has a nontrivial normal subgroup.

15. (a) Suppose that G is the internal direct product of $N_1, N_2, ..., N_m$. Then, for $i \neq j$, $N_i \cap N_j = (e)$ and. If $a \in N_i, b \in N_j$; then $ab = ba$.

Or

(b) If A and B are groups, prove that $A \times B$ and $B \times A$ are isomorphic.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) State and prove Sylow's theorem for abelian groups.

Or

(b) If H and K are finite subgroups of G of orders $o(H)$ and $o(K)$ respectively, then prove that $o(HK) = \dfrac{o(H)o(K)}{o(H \cap K)}$.

17. (a) If G is a finite group and $H \neq G$ is a subgroup of G such that $o(G) \nmid i(H)!$, then H must contain a non-trivial normal subgroup of G.

Or

(b) State and prove Cayley's theorem.

18. (a) State the prove Cauchy's theorem.

Or

(b) Prove that S_n has a normal subgroup of index 2, the alternating group A_n, consisting of all even permutations.

19. (a) State and prove the second part of Sylow's theorem.

Or

(b) State Sylow's theorem and give the third proof.

20. (a) Prove that every finite abelian group is the direct product of cyclic groups.

Or

(b) Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.